Azbel'–Kaner Cyclotron Resonance in Gold*

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The results of a study of Azbel'–Kaner cyclotron resonance in gold are reported. Resonances were observed for all orientations of applied magnetic field in the (110) crystallographic plane, but identification of the observed masses with specific orbits was limited to angular regions near the principal crystallographic axes because of a relatively small experimental $\omega\tau$. Partial agreement was found between the masses observed with magnetic field parallel to principal crystallographic axes and masses obtained by Shoenberg from de Haas–van Alphen measurements. Resonances were also observed with the magnetic field tipped out of the sample surface plane at small and large angles. Differences between the effects observed at small angles and those observed by other workers in copper and silver are discussed. The resonances observed at large angles are identified with orbits. The angular ranges over which these resonances were observed provide additional confirmation of the correctness of the Fermi-surface neck dimensions established by de Haas–van Alphen and magnetoacoustic geometric resonance experiments.

I. INTRODUCTION

THE Fermi surfaces of the noble metals copper, silver, and gold have been the objects of intensive experimental study in recent years. Among the experiments which yield detailed information about the properties of electrons at or near the Fermi surface of a metal is Azbel'–Kaner cyclotron resonance. Cyclotron resonance has been observed and studied in copper[1,2] and in silver.[3] We report here the results of an experimental investigation of cyclotron resonance in gold.

The Fermi surfaces of copper, silver, and gold are similar and consist of distorted spheres with contacts at the ⟨111⟩ faces of the Brillouin zone. The evidence for this Fermi surface form in gold comes from a variety of experiments. The topology of the Fermi surface has been determined from a study of transverse magnetoresistance by Gaidukov.[4] A detailed study of the de Haas–van Alphen effect has been made by Shoenberg.[5] The magnetoacoustic geometric resonance has been studied by Bohm and Easterling.[6] The results of all of these experiments are in good agreement on the general and detailed form of the Fermi surface in gold. Roaf[7] has fitted an analytic expression for the Fermi surface to Shoenberg's de Haas–van Alphen results. After a brief description of the experimental aspects of the present investigation in Sec. II, we discuss in Sec. III the observed masses in terms of the Roaf-Shoenberg Fermi surface.

In the course of these experiments, it was found that in some situations the resonance changed markedly if the applied magnetic field was tipped at a small angle with respect to the sample surface plane. A variety of such "small-angle tip effects" have been observed in other materials, e.g., tin,[8,2] aluminium,[9] sodium and potassium,[10] copper,[2] and silver.[3] Although a detailed theory which can account for all of the effects observed has not yet appeared, several mechanisms which successfully explain some of them have been discussed by Koch, Stradling, and Kip[2] and by Grimes and Kip.[10] For the particular case of aluminum, Grimes *et al.*[9] have reported a detailed calculation based on one of these mechanisms which agrees closely with experiment. In Sec. IV we compare our observations in gold with those in other materials and discuss them in relation to the proposed mechanisms.

We also observed Azbel'–Kaner-type resonances when the magnetic field was tipped at large angles with respect to the sample surface plane. Such large-angle resonances were first reported in tin by Langenberg, Kip, and Rosenblum[11] and have recently been studied in some detail in tin by Koch and Kip,[12] in bismuth by Everett,[13] in copper by Koch, Stradling, and Kip,[12] and in silver by Howard.[3] Our large-angle resonance results are discussed in Sec. V.

II. EXPERIMENTAL DETAILS

Data was taken using a microwave-reflection spectrometer operating at 70 Gc/sec. The sample formed

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the end wall of a cylindrical cavity resonator in which the degeneracy of the two orthogonal TE_{111} modes had been removed by a pertubation of the cavity wall. This made possible the use of either of two cavity modes, one with the rf current polarized parallel to the applied magnetic field, the other with rf current polarized perpendicular to the magnetic field. The cavity was coupled to the side wall of the waveguide in the manner described by Koch and Kip[12] so that the angle between the sample surface and the applied magnetic field could be varied by rotating the magnet. Orientation of the magnetic field with respect to the crystallographic axes of the sample was accomplished by rotating the sample about the cavity axis. These two degrees of rotational freedom permitted the magnetic field direction to be varied over a full 4π solid angle with respect to the sample crystallographic axes. This geometry also permitted variation of the cavity coupling using a moveable short circuit in the waveguide, a feature useful in optimizing the spectrometer sensitivity. The klystron frequency was locked to the resonant frequency of the experimental cavity and magnetic field modulation was used so that the detected signal was proportional to the derivative with respect to magnetic field of the sample surface resistance.

All experiments were done at 4.2°K; the quality of the resonances did not improve perceptibly when the temperature was lowered to 1.5°K.

The sample used was cut from a single crystal used by Bohm and Easterling[6] in their magnetoacoustic geometric resonance experiments. The sample surface was within 1° of a {110} plane. The surface was originally prepared by heavily electropolishing the original lapped surface of the crystal. The same surface was later spark-planed and repolished in an effort to improve the signal. The resistivity ratio [ρ(300°K/ρ(4.2°K)] of the sample was measured using the eddy-current decay method and was about 750. The surface flatness was checked by observing optical interference in the space between the sample surface and an optical flat using a Vickers projection microscope. This measurement showed undulations about the mean surface plane of about 15′ of arc.

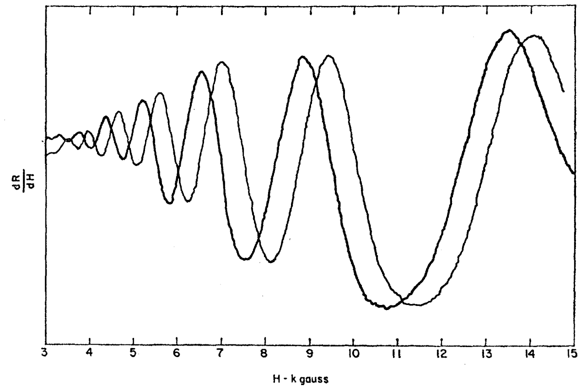

FIG. 1. Derivative signal observed with applied mganetic field in the plane of the sample and near the [001] axis. The light curve was recorded with increasing magnetic field, the heavy curve with decreasing magnetic field. The relative shift of the two curves resulted from use of a relatively long time constant in the detection system with a fast field sweep. The peak near 14 kOe is the $n=2$ resonance subharmonic.

III. CYCLOTRON-MASS RESULTS AND THE FERMI SURFACE OF GOLD

The results discussed in this section were obtained with the applied magnetic field accurately parallel to the sample surface and with the rf current polarized perpendicular to the magnetic field. The field was aligned with respect to the surface by observing a small-angle tip effect in an orientation where it was particularly strong and setting the field at the angle about which the tip effect was symmetrical. Some data were taken with the rf current polarized parallel to the field, but in general the signal-to-noise ratio was low in this mode and it yielded no reliable information not already obtainable from the perpendicular polarization data.

Figure 1 shows one of the better experimental traces obtained. It will be obvious to those familiar with the results of cyclotron-resonance experiments in other materials that the number of subharmonic oscillations observable is smaller than would be desirable for interpretation of the results in terms of the rich variety of cyclotron orbits possible in gold. The apparent $\omega\tau$ in our experiments was about 5, a value quite consistent with the relatively low resistivity ratio of the sample. We have observed resonances for all orientations of magnetic field in the (110) plane. The portion of our data which corresponds to angular ranges where several types of orbits were expected to contribute strongly to the observed resonance did in fact give evidence of the presence of several subharmonic series. Because of the rather small $\omega\tau$, however, it was impossible to resolve these series with any degree of confidence. We have therefore chosen to study carefully and analyze only the data in which the resonance appeared to be a clean, single orbit-type Azbel'-Kaner resonance.

Figure 2 shows the measured cyclotron effective masses as a function of the angle in the (110) plane between the magnetic field and the [001] direction. The masses were determined by making a least-squares fit of a straight line to a plot of the reciprocals of the magnetic fields at which the derivative peaks occurred versus the integers corresponding to the peaks. The normalized effective mass was then calculated using the relation $m^*/m_e=e/m_e c\omega\Delta(1/H)$, where $\Delta(1/H)$ was the slope of the straight line and ω was the rf frequency. The phase shift, defined as the intercept of the straight line on the vertical axis divided by $\Delta(1/H)$, was also calculated, as were the probable errors in both mass and phase shift.

Calculations based on the Azbel'-Kaner theoretical

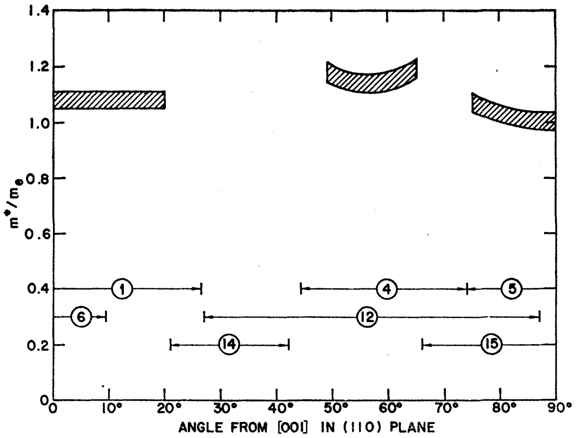

FIG. 2. Cyclotron effective mass versus angle in the (110) plane between the applied magnetic field and the [001] direction. The widths of the bands represent the estimated error of ±0.03. At the bottom are indicated the angular ranges of some of the expected orbit types predicted by the Roaf-Shoenberg Fermi surface. The numbering is the same as that in Fig. 3.

expression for the surface resistance for a spherical Fermi surface show that for $\omega\tau=5$, the mass obtained by analyzing the data in this manner is 0.5% higher than the actual mass and the phase shift is -0.09.[1] For increasing $\omega\tau$, the error in the mass and the phase shift tend to zero. All phase shifts associated with the masses plotted in Fig. 2 were equal to -0.09 to within experimental error. A typical experimental error in phase shift was ±0.05. In these experiments, therefore, with the magnetic field parallel to the sample surface no phase shifts were found which were not expected on the basis of the Azbel'-Kaner theory. No correction was made for the 0.5% error in mass which resulted from the method of analysis since it was negligible compared with the total estimated experimental error. The data of Fig. 2 represent the results of many runs under different conditions. The estimated error was based primarily on the scatter of the experimental points.

Between 0° and 20° the resonance signal had the character of the theoretical derivative curve (dR/dH) for a single carrier with $\omega\tau=5$, or slightly larger; no distortion of the signal from this curve was apparent. In general, the signal was observed to the fifth subharmonic peak, except for the region near 10° where the eighth subharmonic was detected. The mass remained constant to within the scatter of the experimental points from 0° to 20° and was 1.08±0.03. In the range from 20° to 50°, distortions of the resonance appeared which indicated the presence of several types of orbits with different masses which could not be clearly resolved. Between 49° and 64°, the resonance had the characteristics of a slightly distorted single mass resonance. At 55°, the mass was 1.14±0.03 and increased slowly toward smaller and larger angles. Between 65° and 75°, the resonance again had a complicated unresolved structure. Between 75° and 90°, a simple resonance was observed corresponding to a mass which slowly decreased to 1.00±0.03 at 90°.

The orbit types corresponding to these masses were identified using the Roaf-Shoenberg Fermi surface. Figure 3 shows a projection of this surface onto the (110) plane in an extended zone space. The angular ranges of the principal types of cyclotron orbits are indicated on this figure and some are reproduced at the bottom of Fig. 2. The isotropy and angular range of the mass observed near the [001] direction indicates that this mass corresponds to the ⟨100⟩ belly orbits (1). The four-cornered rosette orbit (6) might be expected to contribute at angles less than 9°, but there was no significant difference between the resonances inside and outside the range of this orbit, nor were there any distortions of the resonance line shape which might have indicated the presence of a second orbit type. We therefore must conclude either that the rosette-orbit resonance had too small an amplitude to be observable or that the rosette orbit has a mass very nearly equal to the ⟨100⟩ belly mass.

The mass observed between 49° and 64° undoubtedly corresponds to the ⟨111⟩ belly-type orbit (4). Aside from multiple-zone extended orbits, the only other orbit type which exists in this range is the neck orbit (12). Shoenberg[5] has measured a neck mass of 0.44; we were unable to resolve any resonance corresponding to a mass near this value.

The mass observed near the [1̄10] direction would appear at first to be that of the dog's-bone orbit (5). However, Koch, Stradling, and Kip[2] found in copper another resonance in this range with a mass differing by a few percent from the dog's-bone mass and with roughly the same angular dependence. They identified this mass with the noncentral belly orbit (15). That a contribution from this orbit is present in our data is suggested by the fact that the subharmonic resonances

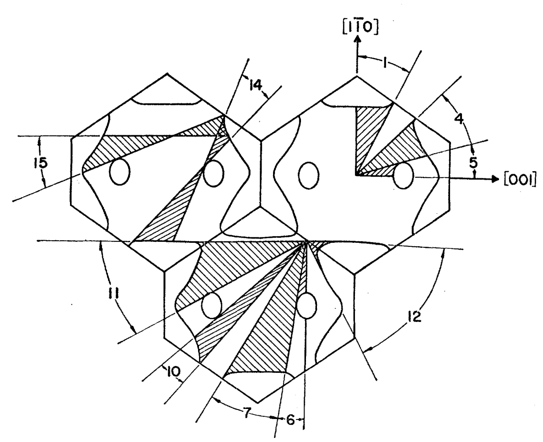

FIG. 3. The Fermi surface of gold projected onto the (110) plane in an extended zone space with the angular ranges of the principal types of cyclotron orbits indicated. To facilitate comparison the numbering of the orbits follows the scheme of Koch, Stradling, and Kip.[2]

do not gradually decrease in amplitude with decreasing field as they do for other orientations, but cut off quite sharply at about the fourth subharmonic. This is just the behavior to be expected from two superimposed resonances corresponding to slightly different masses. The behavior of the resonance at angles less than 75° also suggests the presence of the noncentral belly orbit. The observed mass must therefore be interpreted as an average of the dog's-bone orbit mass and a nearly equal noncentral belly orbit mass. Further work with improved samples may permit resolution of these two resonances.

No clear evidence was found for any extended orbits with masses near twice the free-electron mass, such as orbits (7), (10), and (11) in Fig. 3.

The masses observed with magnetic field along the principal directions are given in Table I. The masses

TABLE I. Cyclotron masses in gold.

Magnetic field direction	Orbit type	Cyclotron mass m^*/m_e Present experiment	de Haas–van Alphen[a]
[001]	⟨100⟩ belly	1.08±0.03	1.19±0.03
	four-cornered rosette		1.09±0.07
[1$\bar{1}$1]	⟨111⟩ belly	1.14±0.03	1.09+0.10
[1$\bar{1}$0]	dog's bone	1.00±0.03[b]	0.98±0.1
			1.00±0.1

[a] See Ref. 5.
[b] See text for discussion of possible contribution of noncentral belly orbit to this mass.

obtained from de Haas–van Alphen measurements by Shoenberg[5] are given for comparison. The agreement between the two sets of masses is satisfactory for the dog's-bone and ⟨111⟩ belly orbits, but is not good for the ⟨100⟩ belly orbit where Shoenberg quotes his smallest error. Shoenberg's ⟨100⟩ rosette mass matches almost exactly the mass we have interpreted as the ⟨100⟩ belly mass. It is tempting to suggest that we have actually observed the rosette but we have ruled out this possibility for the reasons given above. A further argument against this actually being the rosette mass is the difficulty experienced by other workers[2,3] in resolving the rosette mass in the (110) plane in cyclotron-resonance experiments on copper and silver. In the present experiments, with a similar Fermi surface and much lower $\omega\tau$ we could probably not expect to see the rosette resonance.

IV. SMALL-ANGLE TIPPING RESULTS

We report in this section the results of an investigation of the effect of tipping the applied magnetic field at small angles with respect to the sample surface plane. The need for careful control of this tip angle first became apparent when we encountered a lack of reproducibility between experimental runs using a geometry in which the tip angle could not be precisely varied or controlled. This was found to be due to run-to-run variations of less than a degree in the angle between the field and the sample surface. It has since been amply demonstrated in the present investigation and in the work of others[2,3,9,10] that the tip angle is a very important experimental parameter indeed.

In discussing the results of tipping experiments the crystallographic axis referred to is the direction of the magnetic field when it is accurately parallel to the sample surface. The field was tipped in the plane defined by the given axis and the normal to the sample surface.

When the field was tipped about the [1$\bar{1}$1] axis the resonance was found to be insensitive to tipping. There was no appreciable reduction in amplitude or change in character of the resonance up to 5° tip. Beyond 5° tip the resonance amplitude decreased and the resonance became undetectable at about 10° tip. When the field was tipped about the [1$\bar{1}$0] axis the resonance amplitude decreased rapidly and the resonance disappeared at about 1°. The resonance peaks shifted to higher values of magnetic field with increasing tip angle in such a way that the cylotron effective mass remained constant to within our experimental accuracy but the phase shift decreased from -0.05 at zero tip to -0.20 at 1° tip.

The most interesting behavior occurred when the field was tipped about the [001] axis. Figure 4 shows the variation with tip angle of the magnetic fields at which the second, third, and fourth subharmonic derivative maxima occurred. For all orientations investigated any changes in the resonance on tipping were found to be symmetric about the angle at which, as nearly as we could determine, the magnetic field was parallel to the sample surface. This angle was taken to be the zero-tip angle. The data from which Fig. 4 was taken followed the resonance to extinction only for positive tip angles and did not extend above 15 kOe, but other data showed the form of the shift of the second subharmonic peak above 15 kOe to be similar to that of the third and fourth. Figure 5 shows the masses and phase shifts for tipping about the [001] axis. The mass

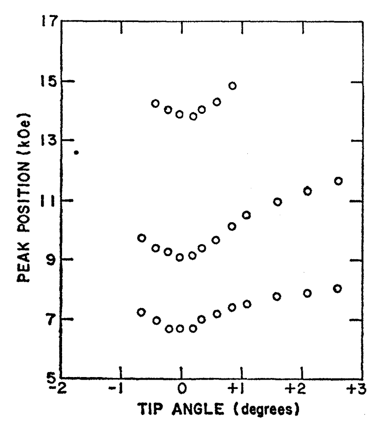

FIG. 4. Derivative peak shift with tip angle for tipping about the [001] axis. The magnetic fields at which the second, third, and fourth derivative peaks occurred are shown.

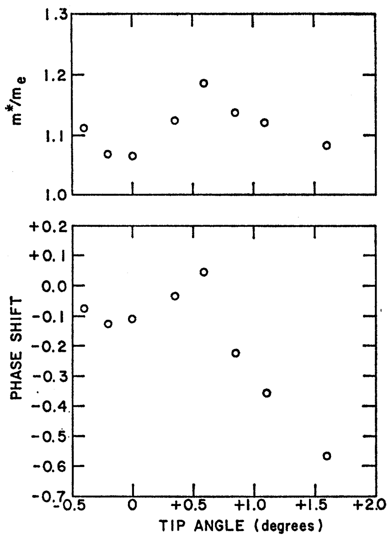

FIG. 5. Cyclotron effective masses and phase shifts versus tip angle for tipping about the [001] axis. The plotted points were calculated from the data of Fig. 4.

and phase shift vary together and nonmonotonically with tip angle, first increasing and then quite sharply decreasing with increasing angle.

The degree of flatness of the sample surface is an important factor in assessing the effects of small-angle tipping. The Doppler-shift mechanism discussed below predicts resonance peak shifts which are linear in tip angle. The range of angles around zero tip over which the peak shift is nonlinear can therefore be taken as a measure of the deviation from perfect flatness of the surface. It is very difficult to make a case for a linear dependence in the data of Fig. 4, but if the curves are extrapolated from the larger angles back to zero assuming that the sections near zero tip should be linear, a roughness of about 15′ can be estimated. This value agrees with the estimate from the optical interference measurement mentioned in Sec. II.

The similarity of the Fermi surfaces which have been established for copper, silver, and gold would suggest that similar tipping effects should be observed in all three materials. A comparison of the present results with those in copper[2] and in silver[3] shows that this is only partly so. The resonances are found to be insensitive to tipping about a ⟨111⟩ axis in all three materials. On tipping about a ⟨110⟩ axis in copper, Koch, Stradling, and Kip (KSK) found that the resonance peaks shifted linearly with tip angle out to 1° but found that the fractional shift was not the same for all peaks of a subharmonic series. The peaks for nonzero tip angle were therefore not periodic in the reciprocal of the magnetic field, and the shift could not be described in terms of a change in cyclotron effective mass. Howard found a more complicated behavior on tipping about a ⟨110⟩ axis in silver. The resonance peaks were observed to split as the field was tipped and then to become inverted, with minima occurring near the points where maxima occurred at zero tip angle. Similar splitting and inversion were found for tipping about a ⟨100⟩ axis in both copper and silver. KSK found that the fractional shift of the split peaks was again significantly different for different subharmonics, whereas Howard found that to within his experimental accuracy, the fractional shifts were the same and his results could be described in terms of changes in mass. We observed only a shift of the resonance peaks with no indication of any splitting or inversion of the peaks for tipping about either a ⟨100⟩ or a ⟨110⟩ axes. The resonance continued to be periodic in the reciprocal of the magnetic field for all angles of tip to within the scatter of the experimental points. It should be noted, however, that because we were only able to observe an appreciably smaller number of subharmonics than either KSK or Howard, our scatter is larger than theirs and may obscure small differences in fractional peak shift from one subharmonic to another.

Two different mechanisms have been proposed to explain the varied effects which occur on tipping. One invokes the Doppler-shifted electromagnetic field seen by an electron which is moving with respect to the source of the field, and has been discussed in some detail by KSK. It is based on the fact that if the magnetic field is not precisely parallel to the sample surface, an electron which has a nonzero average velocity parallel to the magnetic field ($\bar{v}_H$) will reach the top of its orbit at a different depth in the metal on each orbit traversal. Provided $\bar{v}_H$ or the tip angle is sufficiently small so that the electron does manage to pass through the skin several times, the phase of the rf field which the electron sees on successive traversals depends not only on the relative magnitudes of the rf frequency and the cyclotron frequency, but also on the depth at which it makes each traversal of the skin. This is because the rf field changes phase as it propagates into the metal, i.e., the rf field has a wave vector with a real part. The condition for resonance for that particular electron therefore depends on its $\bar{v}_H$ and the tip angle as well as ω and ω_c. A simple calculation using an approximate relation for the rf field in the skin gives a fractional shift in the resonance frequency or resonance field for small tip angles

$$\Delta H/H \cong \bar{v}_H\theta/\delta\omega\,, \tag{1}$$

where $\bar{v}_H$ is the component of the electron's velocity parallel to the magnetic field averaged over an orbit, θ is the tip angle, ω is the rf angular frequency, and δ is a measure of the depth of penetration of the rf field.

If an argument can be made that all of the electrons which contribute significantly to a resonance have nearly the same $\bar{v}_H$, as for example the electrons near a particular elliptic limiting point on the Fermi surface, this simple picture predicts a linear peak shift of about the size experimentally observed. For a magnetic field parallel to a ⟨110⟩ or ⟨100⟩ axis in any of the noble metals, however, the electrons which are believed to dominate the resonance behavior are near area extrema on the Fermi surface and have an extremal cyclotron effective mass and velocities $\bar{v}_H$ ranging from negative values through zero at the extrema to positive values. The simple Doppler-shift picture would predict a range of positive and negative shifts for these electrons which would result simply in a broadening out of the resonance unaccompanied by a peak shift. Without the aid of some additional velocity selecting mechanism the linear shift observed by KSK in copper with $\mathbf{H}\|\langle 110\rangle$ and the nonlinear shift we observed in gold with $\mathbf{H}\|\langle 100\rangle$ are inexplicable. KSK have pointed out that the splitting of the peaks which they observe in copper with $\mathbf{H}\|\langle 100\rangle$ might be explained using the Doppler-shift picture if there exist mass extrema at positive and negative nonzero values of $\bar{v}_H$. If electrons near these mass extrema make the major contribution to the resonance a splitting of the resonance at a rate determined by the $\bar{v}_H$ at the mass extrema should be observed on tipping. If this explanation is correct for copper it should also be correct for gold in view of the similarity of the Fermi surfaces. Our failure to observe any sign of a splitting then becomes rather puzzling. We could have detected a fractional splitting of several percent; Eq. (1) would predict a splitting of the order of 25% per degree or a total of more than 100% over the range in which we were able to observe the signal.

The other mechanism which has been proposed to explain tipping phenomena involves a two-step process in which electrons which have interacted once with the rf field in the skin then orbit into the interior and generate layers of current and field in the interior. Virgin electrons orbiting up toward the surface interact with these fields and reach the surface with drift velocities which are coherent in phase with the drift velocities of the original electrons and hence can contribute to the surface current in a resonant manner. This mechanism has been used successfully by Grimes *et al.*,[9] to explain the splitting, inversion, and apparent mass doubling of the resonance of the second-zone hole Fermi surface in aluminum. Its applicability in that case depended on a special feature of the second zone surface, the fact that it is effectively composed of a set of spherical caps or elliptic limiting points on which all of the effective electrons have nearly the same (large) $\bar{v}_H$. Grimes and Kip[10] have suggested that this mechanism might account for the similar splitting and inversion (with no mass doubling) observed in sodium and potassium, and KSK and Howard have also discussed this mechanism as a possible explanation of their observations in copper and silver. A consideration of the dynamics of electrons in slightly tipped magnetic fields indicates that if this mechanism, rather than the Doppler shift, is responsible for the splitting and inversion the effect ought to appear whenever the dominant electrons are on a part of the Fermi surface which does not have too large a variation in cyclotron mass over a reasonably large range of V_H. The similarity of the Fermi surfaces of copper, silver, and gold would again lead us to expect that these conditions should be satisfied on all three at least for $\mathbf{H}\|\langle 100\rangle$ and that we should have observed a splitting and inversion for this orientation. The absence of any sign of such an effect in our experiments might then be interpreted either as an indication that the masses in gold vary appreciably more rapidly over the Fermi surface than they do in copper and silver or that the splitting and inversion occurs only at quite high values of $\omega\tau$. The former conclusion does not seem consistent with the fact that our mass anisotropy is small and very similar to that found in the other two materials. There is also the possibility that our observations on tipping about the [001] axis result from interference between resonances from two types of orbit, e.g., the $\langle 100\rangle$ belly and rosette orbits. An experimental check of these possibilities awaits higher-resolution experiments with a better sample.

V. LARGE-ANGLE TIPPING RESULTS

We have studied the behavior of the resonance as the magnetic field was tipped out of the sample plane over a full 90°. The field was tipped in the $(1\bar{1}0)$ plane normal to the sample plane so that for zero tip it was parallel to the [001] axis and for 90° tip it was normal to the sample surface and parallel to the [110] direction.

The behavior for small tip angles has been discussed in the previous section. This signal disappeared at a tip angle of about 5°. At 16° a signal appeared and gradually increased in amplitude, reaching a maximum at 24°, then disappeared quite sharply at 26°. Resonance was again observed between 41° and 43°, with an amplitude maximum at 42° and again between 60° and 74° with an amplitude maximum at 64°. The maximum signal amplitude at 24° was comparable to that observed with field parallel to the sample surface; the others were smaller. At 24° the cyclotron effective mass was 1.18 ± 0.02 and the phase shift was -0.08 ± 0.04.

If a large-angle resonance is observed there must be somewhere on the Fermi surface a band containing an appreciable number of electrons with small $\bar{v}_H$ and a cyclotron mass which is either extremal or varies only slowly with k_H (k_H is the wave-vector component parallel to the magnetic field). Electrons satisfying these conditions can return repeatedly to the skin and contribute to the resonance even for large tip angles. These conditions are obviously satisfied by the $k_H=0$ electrons for magnetic field orientations where the $\langle 100\rangle$ and $\langle 111\rangle$ belly-type orbits can exist. We believe that the large-angle resonances observed between 16° and 26° and between 60° and 74° correspond to $\langle 100\rangle$ and $\langle 111\rangle$ belly orbits, respectively. Figure 6 shows these orbits and the angular range for which they were observed. The relatively sharp cutoff at 26° is due to the

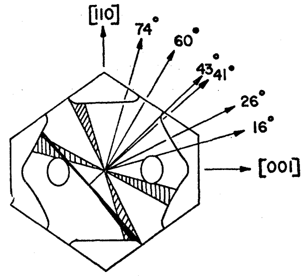

FIG. 6. The angular ranges for which large-angle resonances were observed and the corresponding orbits.

elimination of belly orbits at $k_H=0$ by the necks. This experimentally observed angle is in good agreement with the angle of 26.3° predicted for this cutoff by the Roaf-Schoenberg surface. The fact that the mass observed at 24° near the cutoff angle is considerably higher than the ⟨100⟩ belly mass is consistent with the rapid increase in mass which occurs as cutoff is approached. The disappearance of the signal at 74° is associated with the cutoff by the necks of the ⟨111⟩ belly orbits. The Roaf-Shoenberg model here predicts an angle of 74.0°; again the agreement is good. The resonance observed between 41° and 43° is more difficult to identify. A large-angle resonance for the ⟨111⟩ belly orbit would be expected near the lower limit of the angular range over which this type of orbit can exist. If the observed resonance is attributed to the ⟨111⟩ belly orbit near its lower limit, the neck would have to subtend an angle at the zone center of 12° or less in contrast to the angle of 18.0° of the Roaf-Shoenberg model. The noncentral orbit (14) has a cutoff at 42.0° in the Roaf-Shoenberg model. Inspection of Roaf's curves and a Fermi-surface model based on them indicates that near this cutoff angle there is a range of k_H of reasonably width where $\bar{v}_H$ is small and where an extremal mass may exist. We therefore attribute this large-angle resonance to this noncentral orbit. With this identification, the cutoff angles of all of the large-angle resonances observed agree well with angles predicted by the Roaf-Shoenberg model. This agreement can be taken as additional experimental confirmation of the neck size determined by the de Haas–van Alphen experiments of Shoenberg and by the magnetoacoustic geometric resonance experiments of Bohm and Easterling. The absence of the expected large-angle resonances for the ⟨111⟩ belly orbit near its 44° cutoff remains as a question which may be answered by further work.

VI. CONCLUSION

Within the limitation imposed by the available sample the cyclotron mass anisotropy in the {110} plane appears to be quite similar to that observed for copper and silver. This was only to be expected because of the similarity of the Fermi surfaces for these three metals. The absence of any anomalous phase shifts when the magnetic field is accurately parallel to a relatively flat sample surface corroborates the conclusion of Koch, Stradling, and Kip[2] that the phase shifts observed by Kip, Langenberg, and Moore[1] were the result of small-angle tip effects on an insuffiicently flat sample. The agreement between our cyclotron masses and the masses obtained from de Haas–van Alphen measurements is satisfactory except for the ⟨100⟩ belly mass. The cutoff angles of the large angle resonances which we have observed provide further evidence of the accuracy of the Fermi surface geometry in the region of the necks established by de Haas–van Alphen and magnetoacoustic geometric resonance experiments.

Several problems remain for further study. The great variety of small-angle tipping effects observed in these and other experiments has not yet been fitted into one coherent theoretical and experimental picture. In the particular case of gold, further experiment with better samples are needed to determine whether the absence of any peak splitting and inversion on tipping is a result of a small $\omega\tau$ or of some difference between the Fermi surface of gold and the Fermi surface of copper and silver. Such experiments are now under way. They should also yield more detailed mass anisotropy information and shed some light on such questions as the peculiar nonlinear peak shifts observed in tipping about the [001] axis and the apparent discrepancy between the cyclotron resonance and de Haas–van Alphen ⟨100⟩ belly masses.

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